Provably good task assignment on heterogeneous multiprocessor platforms for a restricted case but with a stronger adversary

Gurulingesh Raravi*, Björn Andersson†* and Konstantinos Bletsas*
*CISTER-ISEP Research Center, Polytechnic Institute of Porto, Portugal
†Software Engineering Institute, Carnegie Mellon University, Pittsburgh, USA
Email: *{ghri, baa, ksbs}@isep.ipp.pt, †baandersson@sei.cmu.edu

***Abstract*—Consider the problem of scheduling a set of implicit-deadline sporadic tasks to meet all deadlines on a heterogeneous multiprocessor platform. We consider a restricted case where the maximum utilization of any task on any processor in the system is no greater than one. We use an algorithm proposed in [1] (we refer to it as LP-EE) from state-of-the-art for assigning tasks to heterogeneous multiprocessor platform and (re-)prove its performance guarantee for this restricted case but for a stronger adversary. We show that if a task set can be scheduled to meet deadlines on a heterogeneous multiprocessor platform by an optimal task assignment scheme that allows task migrations then LP-EE meets deadlines as well with no migrations if given processors twice as fast.**

***Keywords*-heterogeneous multiprocessor, task migrations, real-time scheduling.**

I. Introduction

A heterogeneous multiprocessor platform is a computer system where (i) not all processors are of the same type and (ii) the execution time of a task depends on the processor on which it executes. Many chip makers offer or plan to offer products for computers with different types of processors. The Cell processor is a single chip comprising one main processor (Power4) and eight so-called synergistic processors (optimized for executing SIMD instructions) [2]. NVIDIA (and also AMD) offers general purpose graphics processor units which together with a normal processor are found in most personal computers today [3]. The Intel Sandy Bridge processor [4] is a single chip comprising an x86 multicore processor and a graphics processor. The AMD Fusion processor is a planned single chip comprising an x86 multicore processor and a set of accelerator processors for both embedded platforms [5] and desktops [6]. In a joint effort, ARM and NVIDIA are planning to offer chips comprising one general-purpose and one graphics processor [7]. It is clear that the above mentioned chips are key components in heterogeneous multiprocessor systems and such systems are increasingly used in practice.

An algorithm for deciding whether or not an implicit-deadline task set can be scheduled on a heterogeneous platform exists [8] but it assumes that tasks can migrate. This assumption is often unrealistic in practice, since processors with different functionalities typically have different instruction sets. Thus, the problem of assigning tasks to processors and then scheduling them with a uniprocessor scheduling algorithm (i.e., without migration) is of much greater practical significance. It requires solving two subproblems: (i) assigning tasks to processors and (ii) once tasks are assigned to processors, performing a uniprocessor scheduling on each processor. The latter problem is well-understood (e.g., one may use Earliest Deadline First scheduling [9]) – the difficult part is the task assignment.

The task assignment on a heterogeneous multiprocessor platform is modeled as Zero-One Integer Linear Programming (ILP) in [1][10]. Such a formulation can be solved directly but has high computational complexity. In particular, the decision problem ILP is NP-complete and even with knowledge of the structure of the constraints in the modeling of heterogeneous multiprocessor scheduling, no polynomial-time algorithm is known ([11], p. 245). Via relaxation of ILP formulation to Linear Program (LP) and certain tricks [12], better time-complexity can be attained [1][10]. (Polynomial time-complexity for the algorithm in [10] and for the special case of fixed number of processors, the algorithm in [10] has polynomial time-complexity as well). Both approaches [1][10] offer a performance guarantee that if a task set can be scheduled to meet deadlines on a heterogeneous platform by an optimal task assignment scheme that does not allow task migrations then these approaches meet deadlines as well without allowing task migrations if given processors twice as fast.

In this paper, we address the problem of scheduling a set of implicit-deadline sporadic tasks to meet all deadlines on a heterogeneous multiprocessor platform but for a stronger adversary. We consider a restricted case where the maximum utilization of any task on any processor in the system is no greater than one. We use the approach proposed in [1] (for convenience, we refer to it as *Linear Programming with Exhaustive Enumeration*, abbreviated as LP-EE, described in Section III-A), and (re-)prove its performance guarantee for this restricted case but for a stronger *adversary* (i.e., the set of algorithms against which we evaluate the performance of our algorithm) that allows task migrations. We show that, if a task set can be scheduled to meet deadlines on a heterogeneous platform by an optimal task assignment scheme that allows task migrations then LP-EE meets deadlines as well without allowing task migrations (i.e., partitioned scheduling) if given processors twice as fast.

We would like to reiterate that, the claim in this paper is stronger than the previous state-of-the-art approaches [1][10], as the adversary is more powerful since it allows task migrations.

Minimize $\mathbb{U}$ subject to the following constraints :

C1.	$\sum_{j=1}^{m} x_i^j = 1$	$(i = 1, 2, \cdots, n)$
C2.	$\sum_{i=1}^{n} \left(x_i^j \cdot u_i^j\right) \leq \mathbb{U}$	$(j = 1, 2, \cdots, m)$
C3.	x_i^j is a non-negative **integer**	$(i = 1, 2, \cdots, n)$; $(j = 1, 2, \cdots, m)$

Figure 1. ILP formulation – ILP-Feas(τ, Π)

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C3.	x_i^j is a non-negative **real number**	$(i = 1, 2, \cdots, n)$; $(j = 1, 2, \cdots, m)$

Figure 2. LP formulation – LP-Feas(τ, Π)

II. System Model and Assumptions

A. System Model

We consider the problem of scheduling implicit-deadline sporadic tasks on a heterogeneous multiprocessor platform for a restricted case (i.e., when the maximum utilization of a task in the system is no greater than one). The system is specified as follows:

- **Computing Platform (denoted as Π):** The computing platform consists of m processors. A processor is denoted as $\pi_j \in \Pi$, where $j \in \{1, \cdots, m\}$.
- **Task Set (denoted as τ):** The task set comprises n implicit-deadline sporadic tasks (i.e., for each task, its deadline is equal to its minimum inter-arrival time). A task is denoted as $\tau_i \in \tau$, where $i \in \{1, \cdots, n\}$.
- **Utilization (denoted as U):** The utilization of a task τ_i on a processor π_j is given by u_i^j, a non-negative real number.

B. Assumptions

We make the following assumptions:

- **Independent tasks:** The executions of jobs are independent, i.e., they do not share any resources and do not have any data dependency.
- **Migrations:** In our approach, we constrain the system by assuming that the tasks are not allowed to migrate between processors. However, in our adversary, we relax this constraint on the system by allowing jobs to migrate between processors thereby making the adversary more powerful.
- **Task utilization:** The maximum utilization of any task on any processor in the system is no greater than one, i.e., $\forall i, j : u_i^j \leq 1$.
- **No job parallelism:** A job can be executing on at most one processor at any time instant.

III. The Methodology: Linear Programming with Exhaustive Enumeration (LP-EE)

A. Background and Previous Result

We briefly describe the approach proposed in [1] before proceeding to discuss how we intend to use it and (re-)prove its performance for a stronger adversary.

In [1], the problem of assigning tasks to processors has been formulated as Zero-One ILP as shown in Figure 1. Here $\mathbb{U}$ denotes the maximum capacity of any processor that is used and is set as the objective function (to be minimized). $\mathbb{U} \leq 1$ implies that the sum of utilization of tasks assigned to any processor is less than or equal to the available capacity on that processor. The variable x_i^j (referred to as *indicator variable*) indicate the assignment of task τ_i to processor π_j, i.e., $x_i^j = 1$ implies that τ_i is entirely assigned to processor π_j (such tasks are referred to as *integrally assigned tasks*), $x_i^j = 0$ implies that τ_i is not assigned to processor π_j. The first constraint ($C1$) indicates that every task must be assigned to processors. The second constraint ($C2$) indicates that no processor capacity should be used more than $\mathbb{U}$. The third constraint ($C3$) indicates that the indicator variables must be non-negative integers.

Since, ILP is NP-complete, the formulation is relaxed to LP by allowing the indicator variables to be non-negative **real numbers** (instead of just 0 or 1). The relaxed LP formulation is shown in Figure 2. As we can see, the only change in LP-Feas(τ, Π) formulation compared to ILP-Feas(τ, Π) formulation is that, the $C3$ constraint now allows x_i^j variables to take real numbers instead of just 0 or 1. The semantics of the x_i^j variable remain the same, in addition, $0 < x_i^j < 1$ indicates that *fraction* x_i^j of τ_i is assigned to processor π_j (such tasks are referred to as *fractionally assigned tasks*).

Then a two-step algorithm (referred to as LP-EE) is proposed to assign tasks on a heterogeneous platform. The algorithm is as follows:

1) The LP formulation is solved using an LP solver such as IBM ILOG CPLEX [13]). If $x_i^j = 1$ then task τ_i is (integrally) assigned to processor π_j. Using certain tricks [12], it is shown that there exists a solution to LP-Feas(τ, Π) in which all but at most $(m-1)$ tasks are integrally assigned to processors.
2) The remaining at most $(m-1)$ tasks are integrally assigned on the remaining capacity of the processors using exhaustive enumeration.

Finally, the performance guarantee of this algorithm is proven which is stated as Lemma 1 below.

Lemma 1. ***(from Theorem 3 in [1])***
If there is a feasible mapping of a task set τ on a heterogeneous platform Π in which at most half the capacity of every processor is used, then it is guaranteed that LP-EE generates a feasible mapping (as well) of τ on Π.

Note that, the LP-EE algorithm does not make any assumption on the maximum utilization of a task, i.e., it is applicable to a generic case in which the maximum utilization of a task on a processor in the system can exceed one, i.e., $\exists i, j : u_i^j > 1$.

B. New Result

Now, with the knowledge of the algorithm proposed in [1] and its performance guarantee, let us proceed to discuss the approach in which the adversary is migrative.

Lemma 2. *If a task set τ is feasible on a heterogeneous platform Π with task migrations permitted then LP-Feas(τ, Π) gives a solution with $\mathbb{U} \leq 1$.*

Proof: It is shown in Theorem 2 in [1] that if τ is feasible on Π then ILP-Feas(τ, Π) gives a solution with $\mathbb{U} \leq 1$. Since LP-Feas(τ, Π) is a relaxed formulation of ILP-Feas(τ, Π), $\mathbb{U}$ returned by LP-Feas(τ, Π) is no greater than $\mathbb{U}$ returned by ILP-Feas(τ, Π). Hence, the proof. ■

Throughout this paper, we illustrate the concepts with a (randomly generated) running example. Consider a feasible task set τ with seven tasks to be scheduled on a heterogeneous platform Π with three processors. The utilization of tasks on each processor is shown in Table I.

$(\tau_i \downarrow)(u_i^j \rightarrow)$	$\mathbf{u_i^1}$	$\mathbf{u_i^2}$	$\mathbf{u_i^3}$
τ_1	0.087002	0.066455	1.952548
τ_2	1.294308	0.528062	0.906763
τ_3	0.802204	0.488072	1.240208
τ_4	0.448277	1.076216	1.825816
τ_5	0.573124	1.287740	0.982321
τ_6	0.148060	1.933626	0.654599
τ_7	0.331234	1.284164	0.814624

Table I
AN EXAMPLE TASK SET TO ILLUSTRATE CONCEPTS.

Formulating this system as a linear program using LP-Feas(τ, Π) shown in Figure 2 and inputting it to an LP solver, we obtain the solution shown in Table II and $\mathbb{U} = 0.999999$. It indicates that τ is feasible on Π. The values in Table II correspond to indicator variables which indicate the task assignment to processors. For example, τ_1 is integrally assigned to π_2, τ_2 is fractionally assigned to π_2 and π_3, and so on.

$(\tau_i \downarrow)(\pi_i \rightarrow)$	π_1	π_2	π_3
τ_1	0.000000	1.000000	0.000000
τ_2	0.000000	0.843599	0.156401
τ_3	0.000000	1.000000	0.000000
τ_4	1.000000	0.000000	0.000000
τ_5	0.126375	0.000000	0.873625
τ_6	1.0000	0.0000	0.0000
τ_7	1.0000	0.0000	0.0000

Table II
A SOLUTION (I.E., THE VALUES OF x_i^j VARIABLES) BY LP SOLVER TO THE TASK SET SHOWN IN TABLE I.

Now, if we "divide the result in Lemma 2 by 2", i.e., divide the utilization of every task on every processor by a factor of 2 (we refer this new task set as τ'), and divide the speed of every processor by 2 (we refer this new task set as Π'), we get the following result.

Corollary 1. *If a task set τ' is feasible on a heterogeneous platform Π' with task migrations permitted then LP-Feas(τ', Π') gives a solution with $\mathbb{U} \leq 1$.*

Lemma 3. *If LP-Feas(τ', Π') gives a solution with $\mathbb{U} \leq 1$ then LP-Feas(τ', Π) gives a solution with $\mathbb{U} \leq 0.5$.*

Proof: Let us assume that LP-Feas(τ', Π') gives a solution with $\mathbb{U} \leq 1$. To show that LP-Feas(τ', Π) gives a solution with $\mathbb{U} \leq 0.5$ it suffices to show that there *exists* a solution with $\mathbb{U} \leq 0.5$) – then LP-Feas(τ', Π) will output the same solution or a better one (i.e., with even lower $\mathbb{U}$).

Let u_i^j and $u_i^{j'}$ respectively denote the utilizations of a task $\tau_i \in \tau'$ on processor $\pi_j \in \Pi$ and on processor $\pi_j^{'} \in \Pi^{'}$. Then, by definition (of Π and $\Pi^{'}$), it holds that:

$$u_i^j = \frac{u_i^{j'}}{2} \tag{1}$$

Let $x_i^{j'}$ denote the indicator variables for the solution output by LP-Feas(τ', Π'). Let x_i^j denote the indicator variables for the LP-Feas(τ', Π) problem instance. Let us copy the values of x_i^j into $x_i^{j'}$, i.e.,

$$\forall i, j : x_i^j = x_i^{j'} \tag{2}$$

Now, consider the three conditions listed in Figure 2. It is easy to see that conditions $C1$ and $C3$ hold true. Now, let us analyze $C2$. Using Expression 1 and 2, we get:

$$\sum_{i=1}^{n} \left(x_i^j \cdot u_i^j \right) = \sum_{i=1}^{n} \left(x_i^{j'} \cdot \frac{u_i^{j'}}{2} \right) = \frac{1}{2} \cdot \sum_{i=1}^{n} \left(x_i^{j'} \cdot u_i^{j'} \right)$$

We know (by assumption) that $\sum_{i=1}^{n} \left(x_i^{j'} \cdot u_i^{j'} \right) \leq \mathbb{U}$ and $\mathbb{U} \leq 1$. Hence,

$$\frac{1}{2} \cdot \sum_{i=1}^{n} \left(x_i^{j'} \cdot u_i^{j'} \right) \leq \frac{1}{2}$$

Hence, there exists a solution with $\mathbb{U} \leq \frac{1}{2}$ which satisfies $C1 - C3$. Thus, $LP - Feas(\tau', \Pi)$ (which tries to minimize $\mathbb{U}$) definitely finds such a solution. ■

Combining Corollary 1 and Lemma 3, we get:

Corollary 2. *If a task set τ' is feasible on a heterogeneous platform Π' with task migrations permitted then LP-Feas(τ', Π) gives a solution with $\mathbb{U} \leq 0.5$.*

Our example task set (shown in Table I), after dividing by 2, is shown in Table III. Formulating this system as a linear program using LP-Feas(τ', Π) and inputting it to an LP solver, we obtain the solution that is same as the one shown in Table II but with $\mathbb{U} = 0.499999$.

$(\tau_i \downarrow)(u_i^j \rightarrow)$	$\mathbf{u_i^1}$	$\mathbf{u_i^2}$	$\mathbf{u_i^3}$
τ_1	0.043501	0.033227	0.976274
τ_2	0.647153	0.264030	0.453381
τ_3	0.401102	0.244036	0.620103
τ_4	0.224138	0.538108	0.912908
τ_5	0.286561	0.643870	0.491160
τ_6	0.074030	0.966813	0.327299
τ_7	0.165616	0.642082	0.407311

Table III
TRANSFORMED TASK SET OBTAINED AFTER DIVIDING THE ORIGINAL TASK SET (SHOWN IN TABLE I) BY 2.

We know the upper bound on the number of fractionally assigned tasks in the assignment corresponding to the LP solver solution for the LP-Feas(τ', Π') formulation [1]:

Fact 1. *If there are tasks that were fractionally assigned, in accordance with the solution returned by LP solver after*

solving LP-Feas(τ', Π) formulation, then there can be at most $m - 1$ such tasks.

We now combine all the intermediate results to prove the performance of LP-EE (for the restricted case) but for a stronger adversary.

Theorem 1. *If a task set τ' is feasible on a heterogeneous platform Π' with task migrations permitted then LP-EE succeeds in assigning τ' on Π as well (with no task migrations, i.e., partitioned scheduling), where each processor in Π is at most twice faster than the corresponding processor in Π'.*

Proof: Let us assume that τ' is feasible on a heterogeneous platform Π' with task migrations. Now, we show that LP-EE succeeds in assigning τ' on Π. We know that:

O1 **From Corollary 2:** If a task set τ' is feasible on a heterogeneous platform Π' with task migrations permitted then LP-Feas(τ', Π) gives a solution with $\mathbb{U} \leq 0.5$ – all processors in Π are used at most half their capacity.

O2 **From Fact 1:** If there are tasks that were fractionally assigned, in accordance with the solution returned by LP solver after solving LP-Feas(τ', Π) formulation, then there can be at most $m - 1$ such tasks.

O3 **From system model:** $\forall i \in \tau, j \in \Pi : u_i^j \leq 1$. Hence, $\forall i \in \tau', j \in \Pi : u_i^{j'} \leq 0.5$

Hence, the remaining at most $m - 1$ fractional tasks can be integrally assigned in the remaining capacity of the processors, for example, assign each of these tasks to a different processor – this results in a schedulable assignment (follows from $O1$ and $O3$). We can then use EDF [9] to schedule the tasks assigned on each processor.

Since Exhaustive Enumeration looks for all the possible valid assignments for assigning the remaining at most $m-1$ tasks, it succeeds in finding a schedulable assignment (such as the one described above – assign each of these tasks to a different processor).

Hence, the proof. ■

Coming back to our example task set τ', the solution provided by the LP solver (see Table II) has two fractionally assigned tasks, i.e., τ_2 and τ_5. We can see from the solution (ignoring the fractional assignment of τ_2 and τ_5) that π_1, π_2 and π_3 have remaining utilizations of 0.536216, 0.722737 and 1.000000 respectively on computing platform Π (where a processor speed is twice the corresponding processor speed in Π'). Hence, we can assign τ_2 to π_2 and τ_5 to π_1 without violating the EDF schedulability test on any processor.

IV. Summary

We used an existing approach [1] from state-of-the-art for assigning tasks on a heterogeneous multiprocessor platform and (re-)proved its performance guarantee for a restricted case but with a stronger adversary. We showed that if a task set (in which the maximum utilization of a task is no greater than one) can be scheduled to meet deadlines on a heterogeneous platform by an optimal task assignment scheme that allows task migrations then LP-EE meets deadlines as well with no migrations if given processors twice as fast.

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