Dynamic Bandwidth Management in Networked Control Systems using Quantization

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Abstract

Modern networked control systems integrate multiple independent feedback control loops that require guaranteed bandwidth for timely operation. However, planning the distributed control systems considering worst-case requirements leads to expensive and inefficient designs. This motivated the development of dynamic rate adaptation as a technique to support higher integration in these systems while providing an efficient use of the network bandwidth. This bandwidth can also be managed by varying the quantization used in each loop but, surprisingly, to the best of the authors' knowledge, this approach has not been explored yet. In this work-in-progress paper, we propose managing the network bandwidth varying the number of bits used to represent the transmitted variables (sensor readings and actuation values) while keeping the loop rates constant. We present the basics of the quantization-based bandwidth management as well as a qualitative discussion on the pros and cons of this method.

1. Introduction

Embedded systems have generally evolved towards higher distribution motivated, among other factors, by scalability, maintainability, composability and cost requirements [1]. As a result, a growing amount of information is exchanged between system nodes increasing the pressure on the network planning to guarantee timely interactions. This is particularly relevant for distributed control applications for which undesired network delays can cause instability. Thus, the classic design approach of these systems considers the worst-case requirements in terms of amount and rate for each transaction in the system, i.e., control tasks running with constant periods and always producing the same amount of information. Unfortunately, this approach does not promote an efficient use of the system resources, particularly of the network bandwidth, leading to inefficient and expensive designs.

In recent years, the network bandwidth efficiency in distributed computer control systems has improved using flexible approaches that consider average requirements and taking appropriate measures when occasional overloads occur at run time. An example of this is the dynamic rate adaptation technique [2] that adapts the communication rate of distributed feedback loops to the available network and processor bandwidth. It opposes to the overload conditions by reducing the rate of control loops at the expense of a small degradation of the control performance.

Another approach for adapting the communication requirements at run time to control the bandwidth usage is to change the messages size. This can be achieved changing the quantization used in the control loops. Curiously, to the best of the authors' knowledge, this approach has not been explored before in the context of dynamic bandwidth management in distributed control systems despite several apparent advantages.

This paper proposes using quantization for dynamic bandwidth management and compares it qualitatively with rate adaptation. It is a preliminary work that simply explores the concept and opens the way to an experimental validation and quantitative assessment that are underway.

2. Related work

Adjusting sampling rates under overload conditions has been studied for some time. Within a single processor, the works in [3][4][5] use feedback scheduling with LQ controllers to manage the control performance of the loops while enforcing schedulability. In the same scope, the works in [6][7] specify the control tasks with different sets of sampling intervals and time delays, and a specific PID controller for each case. At run time, a scheduler picks the values that guarantee schedulability of the task set and some level of control performance.

The work in [8] addressed networked control systems but following an adaptation approach similar to the one in [6] with a predefined bank of controllers switched online every control iteration according to the sampling to actuation delay. In [9] the original state-space representation of each controlled process is augmented with a state variable describing the network dynamics. The adaptation of the control loops is then done locally avoiding overload conditions. Similarly, the work in [10] addressed overload conditions in distributed control systems where control tasks would adapt their sampling periods according to the network status. Conversely, a centralized manager is proposed in [2], which performs the adaptation of the sampling periods of the control messages and enforces it on the control loops.

All the previous works used rate adaptation to manage control performance and computing or communication bandwidth, disregarding that a similar adaptation could potentially be achieved changing the size of the sampling and control variables by managing the quantizers. On the other hand, most studies on quantization [11][12] did not consider adaptation of the control quality or network bandwidth but just the impact of the quantization errors. More recent works addressed stability and stabilization problems for input and output quantized feedback systems [13][14]. In particular, the work in [15] presented the stability analysis for an output feedback control with finite-level logarithmic quantizers.

More related to our aims are the works in [16], [17] and [18] that already do some form of quantization-based bandwidth management. The former work [16] minimizes the bits of information to be transmitted every control cycle while satisfying a control performance requirement, making use of a so called quantization real-time scheduling scheme. However, it does not explain how the quantization is done. The work in [17] minimizes the use of the communication resources using event-triggered control design of continuous time linear networked systems with quantization. However, it considers infinite logarithmic quantizers. Finally, the work in [18] also combines dynamic quantization and time scheduling, and proves stability using a different concept, namely Input to State Stability (ISS).

We will use the work in [15] as a basis to propose a new way of managing the network bandwidth and control performance using dynamic quantization, with finite-level logarithmic quantizers, which is proved stable under a quadratic stability criterion.

3. Quantization in feedback control loops

In this section we briefly present the effect of using quantizers in control loops. Consider the input and output quantized feedback system in Fig. 1, and the following augmented system which represents the closed-loop system (see [15] for more details):

$$\begin{cases} \zeta(k+1) = A_a\zeta(k) + B_a p(k) \\ q(k) = C_a\zeta(k) \\ p(k) = Q_a\big(q(k)\big) \end{cases} \tag{1}$$

Note that ζ is the closed-loop system state considering plant and controller, and A_a, B_a and C_a the augmented state, input and output matrices, respectively. Q_a aggregates sampling and control quantizers, while $p = [v\ u]^T$ and $q = [y\ w]^T$ are the inputs and outputs vectors.

Both the sampling quantizer $Q_1(\cdot)$ and the control quantizer $Q_2(\cdot)$ are assumed to be logarithmic with finite alphabets following the constructive law in Eq. 2 (i=1,2).

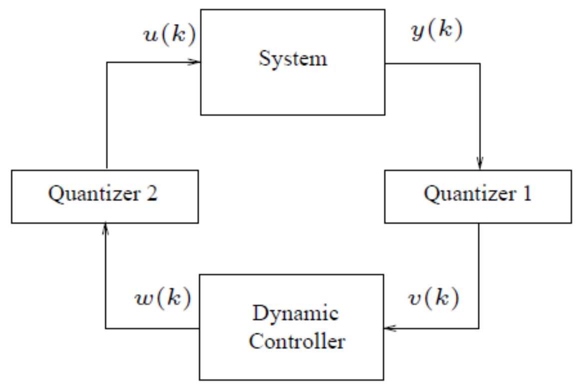

Figure 1. Feedback control system with input and output quantization.

$$Q_i(v) = \begin{cases} \mu_i, \ \text{if } v > \frac{\mu_i}{1-\delta_i},\ \mu_i > 0 \\ {\rho_i}^j\mu_i, \ \text{if } \frac{{\rho_i}^j\mu_i}{1+\delta_i} < v \le \frac{{\rho_i}^j\mu_i}{1-\delta_i}, j = 0,1,\dots N_i - 1 \\ 0, \ \text{if } 0 \le v \le \varepsilon_i \\ -Q(-v), \ \text{if } v < 0 \end{cases} \tag{2}$$

where

$$\delta_i = \frac{1-\rho_i}{1+\rho_i}, \qquad \varepsilon_i = \frac{{\rho_i}^{N_i-1}\mu_i}{1+\delta_i}$$

and N_i is the number of positive quantization levels, μ_i is the largest admissible level and ε_i is the smallest. Note that a small (large) ρ_i implies a coarse (dense) quantization (Fig. 2). As in [11] we abuse the terminology and refer to ρ_i as quantization density. Moreover, note that $0<\rho_i<1$.

Then, according to the closed-loop system in Eq. 1 and the quantizers in Eq. 2, we can introduce two sets $\mathcal{B}_i$ and $\mathcal{C}_i$, i=1,2, which correspond to the largest (μ_i) and smallest (ε_i) quantization levels of quantizer Q_i.

Note that, whenever the state ζ of the system (Eq. 1) is within $\mathcal{C}_i$, it generates $Q_i(q_i) = 0$, which leads to a zero input signal p_i. Hence, the trajectory of ζ will not converge to the system origin (equilibrium point), implying that the closed-loop system asymptotic stability is not ensured.

This behaviour can be tackled in practice using the notion of quadratic stability. We define the set $\mathcal{D}$ of admissible initial conditions, given by $\mathcal{B}_1 \cup \mathcal{B}_2$, and the set $\mathcal{A}$ related with $\mathcal{C}_1 \cup \mathcal{C}_2$ which is an attractor of $\mathcal{D}$. Then, the stability notion presented in [15] ensures that for any $\zeta(0) \in \mathcal{D}$, i.e., any admissible initial condition, the trajectory of the system state $\zeta(k)$ will converge to the set $\mathcal{A}$ within a finite time (Eq. 3).

$$\exists_{\bar{k}>0}: \forall_{k \geq \bar{k}}, \quad \zeta(k) \in \mathcal{A} \qquad (3)$$

4. Quantization and bandwidth management

The relationship between quantization and network bandwidth becomes apparent when considering that different quantizers have different numbers of quantization levels leading to different numbers of bits to represent them. In particular, being N the number of positive quantization levels, the minimum number of bits to represent them is given by Nb according to Eq. 4.

$$Nb = \lceil \log_2 2N \rceil \qquad (4)$$

There are, however, two other important points to consider. On one hand, different numbers of quantization levels must lead to different levels of control performance in a consistent way. Thus, fewer (more) quantization levels lead to fewer (more) bits and worse (better) control performance. On the other hand, the dynamic switching of quantizers should be done in a way that does not impose extra overhead when reconstructing the signals.

These two requirements can be met by keeping the largest quantization level μ and quantization density ρ fixed, consequently δ is fixed, too. Then, just the number of levels N is modified, implying a modification of the smallest quantization level ε according to Eq. 5 (Fig. 2). Remember that ρ<1 and thus, as ε grows, N is reduced.

$$N \geq 1 + \log_\rho \frac{\varepsilon(1+\delta)}{\mu} \qquad (5)$$

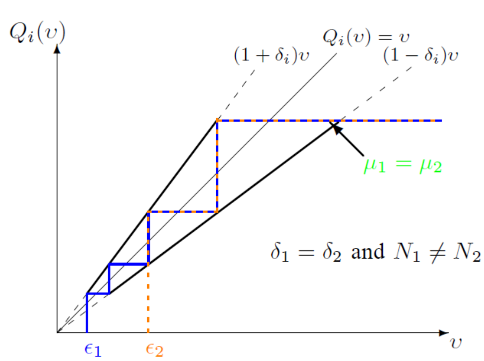

Figure 2. Truncated logarithmic quantizers.

In this case, the set $\mathcal{D}$ of admissible initial conditions remains the same, while the attractor set $\mathcal{A}$ changes (Fig. 3), with its dimension varying consistently with N and inversely to ε. Now, note that the dimension of the attractor set can be used as a metric of control performance since the smaller it is the smoother the system trajectory will be around the equilibrium point. This results directly from Eq. 3 and the notion of stability adopted in this work. According to this notion, the closed-loop system will be stable as long as $\mathcal{A} \subset \mathcal{D}$.

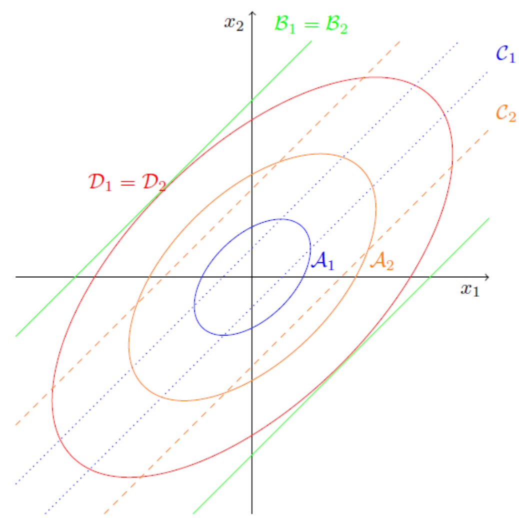

Figure 3. Stability regions of two quantizers.

Therefore, in a dynamic system comprising several independent distributed feedback loops, if sufficient bandwidth is available, these loops can use dense quantizers, reflected in variables expressed with more bits and thus in messages with longer size. If, at a certain point a network overload occurs, it can be mitigated by having a part of the feedback loops moving to coarse quantizers that use shorter messages while keeping stability despite a certain loss in control performance.

Finally, the specific adaptation of the quantizers proposed in this section, in a fixed point representation, corresponds to eliminating least significant bits in the variables that are conveyed over the network. This allows a very simple reconstruction of the values at the controller or actuator, just by detecting the number of bits received and right padding with the necessary 0s to recover the right format.

5. Practicality of the approach

For the proposed method to be effective, the messages size variation must be significant thus with a sufficient impact in the network bandwidth. On the other hand, the typical size of the variables involved is short. In fixed point formats, common sizes will be up to 4 bytes. Thus, the proposed method is only effective in low overhead packet networks, e. g., CAN or FlexRay. In the case of CAN, message sizes of 1, 2 and 4 data bytes lead to worst-case packet bit lengths (all overheads included) of 65, 75 and 95 bits, respectively, which corresponds to a reduction of 21% to 32% in the message durations when moving from 4 to 2 or to 1 bytes, respectively. Moving from 2 to 1 bytes, yields a reduction of 13%.

From the control programs point of view, the adaptation to variable quantizers is straight forward since the receivers know how many bytes are received and thus the received values format can be easily adjusted.

When comparing with rate adaptation, one aspect seems particularly relevant and with potential for significant benefits. In fact, the sampling periods of the control loops remain constant. On the network side, this may favor significant improvements in the temporal behavior of the traffic since more favorable periods, e.g., harmonic, can be used with significant reductions in network delay jitter. On the control side, using constant periods simplifies the controllers design and avoids controller switching with all its associated potential problems and difficulties.

Nevertheless, this technique is not free from concerns since the stability model might not be adequate to some plants that require very low output jitter. In this case, it might not be possible to switch to fewer quantization levels, thus eliminating the potential benefits of this technique. Moreover, quantization adaptation seems to be significantly constrained by the fact that current networks use byte-boundary data fields, thus limiting the resolution of the control over network bandwidth, which becomes coarser than with rate adaptation.

6. Conclusions and Future Work

Many modern distributed embedded systems involve several independent feedback loops which, if designed under worst-case assumptions, become rather inefficient in terms of resource usage, particularly network bandwidth. Previous approaches to alleviate this situation considered rate adaptation. Conversely, in this paper we propose a new direction, exploring the use of quantization to vary the size of the sampling and control variables transmitted over the network. We presented the quantization basics and the stability criterion and we showed that this bandwidth adaptation mechanism is practical and can be easily implemented. Finally, we briefly compared qualitatively with rate adaptation and found interesting trade-offs that need further investigation. Our approach favors the network traffic planning and the controllers design, while the rate based approach gives a fine control over network bandwidth and uses a traditional stability criterion. We are currently implementing the quantization-based approach to gather more insight over its actual benefits.

7. Acknowledgements

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