Probabilistic Component-Based Analysis for Networks

Invited paper
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ABSTRACT

Time-constrained networks have demanded so far for deterministic modeling and analysis in order to guarantee their worst-case behavior. With this work we intend to apply both probabilistic modeling and probabilistic analyses to investigate such networks. The probabilistic framework we propose aims at guaranteeing confidence levels, in the form of probabilities, to the network timing constraints; the deterministic case remain a particular case, the worst-case, within the probabilistic framework. We focus on probabilistic bounds for defining probabilistic interfaces to network components and we study the way that probabilities propagate within networks by accounting for the dependences and the interactions between network components. Finally, we define and apply probabilistic performance metrics for evaluating network behavior with different degree of confidence due to the probabilities.

1. INTRODUCTION

Within todays embedded systems plenty of functionalities and non-functionalities coexist participating defining the system correct execution. Due to that, system interactions have reached a high complexity such that deterministic worst-case bounds become overly pessimistic in modeling system behavior. The pessimism of both modeling and analysis from the worst-cases, together with the resource over-provisioning for accommodating all the timing constraints that systems demands are not affordable anymore. Nonetheless, worst-cases remain fundamental for providing safety guarantees and certification.

Networked embedded systems exasperate the increasing complexity trend for embedded systems, since they compose of multiple elements, included the communication network, any of which may be unpredictable or extremely costly to be made predictable. Even highly timed-constrained networks such as Avionics Full-Duplex Switched Ethernet (AFDX) can have unpredictabilities at their inputs and/or variabilities arising from the network components, e.g. the AFDX switches. The dependences and the interactions between network elements as well as messages accessing them exploit network unpredictabilities. All of that makes deterministic models too pessimistic for the networks. The solution is to accurate model unpredictabilities and to characterize networks behavior while reducing the over-provisioning: the solution demands for more flexibility than deterministic models.

Uncertainties and variabilities make networks resemble probabilistic processes with probability laws at describing their behavior. Being closer to what really happens, probabilistic models are less pessimistic than deterministic ones based only on worst-cases. Moreover, probabilistic models better cope with the different criticality levels that safety-critical networked systems might require. Out of the probabilistic modeling it has to be developed probabilistic analyses. As there exist different time-critical constraints within networks, the use of probability has to face that and provide an edge with respect to the classical deterministic analyses. The probabilistic analysis have to be able to account for all the probabilistic effects while providing the accuracy and the flexibility to guarantee system timing constraints.

Contributions. In this paper we *formalize* the component-based probabilistic analysis approach which faces the natural variable behaviors within networks, e.g. the variability of the message activations or the message payloads (message lengths), the variability of the resource provisioning (bandwidth) to transmit messages, and the queue behavior variability. The probabilistic analysis approach *makes use of the probabilistic models* for either message parameters (activations or payloads/lengths) or network component parameters (bandwidth or queue) might have. The probabilistic analysis extends the notion of bounding curves with probabilities defining *probabilistic bounds*, where the probability expresses the confidence there is on the bound. Then, it applies probabilistic comparisons between bounds to *guarantee timing constraints* at both component and whole network levels. The probabilistic analysis is component-based as it considers the system composing component with their probabilistic representation (component/interfaces). Interesting and challenging are *the guarantees to the network behavior that can be provided with the probabilities.* With our probabilistic framework we are able to provide both hard and soft guarantee to the network component timing behavior as well as to the whole network timing behavior.

The probabilistic network modeling and analysis framework we formalize and we apply hereby is named probabilistic Calculus (pC).

Organization of the paper. In Section 2, we introduce the probabilistic modeling to networks tackling with both message flows and resource provisioning with probabilistic bounds. Section 3 defines the basics for modeling network components as probabilistic components. It presents also the algebra for composing probabilistic bounds (bounding curves and probabilities) within the networks. It describes also how to model a FIFO queue network component with probabilistic bounds. In Section 3 the probabilistic performance metrics are formalized. Section 4 illustrates the theoretical framework and the potential of probabilistic analysis to networks with a test case example. Finally, Section 5 is for conclusions and future work.

1.1 State of the Art

The Network Calculus (NC), [1], is widely applied to investigate networks. It makes use of deterministic bounds to characterize both message flows and network elements timing behavior. Although pessimistic, the NC guarantees certifiable performances of time-constrained avionic networks such as AFDX, [2, 3].

The Real-time Calculus (RTC) [4] instead, tackles with tasks and scheduling elements offering bounds to their timing behavior. Both NC and RTC have algebras to compose bounds and provide behavior analyses to either component-based networks or component-based task execution systems. For the sake of comprehension, in this work we consider NC and RTC equivalent in computing bounding curves, although there are significant differences between their algebras; see [5] for an insight on the NC and RTC differences.

Recently, probabilistic instances of Network Calculus and Real-Time Calculus have been introduced for the modeling and the analysis of realistic networks, where variabilities are the normal behavior. Both the Stochastic Network Calculus (SNC), [6, 7] and the probabilistic Real-Time Calculus (pRTC), [8, 9, 10] contribute to probabilistically modeling flows and resource provisioning with probabilistic bounding curves. In particular, few works make use of SNC for studying networks, focusing on specific system parameters such as delays and backlogs, [11, 12, 13]. With this work we apply both SNC and pRTC basics to define the pC for network modeling and analysis that we apply for investigating timed-constrained embedded networks from probabilistic models of message flows. As we stress the probabilistic nature of message flows, we apply probabilistic analysis to the probabilistic component-based representation of the networks. The flexibility brought by the probabilities allows for more accurate network timing behavior characterization than deterministic approaches, and the possibility of handling both hard and software timing constraints.

2. PROBABILISTIC MODELING BACKGROUND

A message flow can be seen as consecutive message instances which repeat periodically while exhibiting variabilities at both arrival time or payload/length per instance. Each message parameter, due to variabilities from possible sources of uncertainties, can be represented with a random variable[1]. The probabilistic representation of a message m_i is through a distribution $\mathcal{M}_i$ and its associated probabilistic distribution function (pdf)[2] $\mathsf{pdf}_{\mathcal{M}_i}$ which describes the variability of m_i. In case of discrete distribution it is:

$$\mathsf{pdf}_{\mathcal{M}_i} = \sum_{k=0}^{k_i} P(\mathcal{M}_i = x_i^k)\delta_{x=x_i^k} \quad (1)$$

where $P(\mathcal{M}_i = x_i^k)$ is the probability of having value x_i^k for the message parameter (either arrival or payload) within the execution streamline. In the following, this will be noted as :

$$\mathsf{pdf}_{\mathcal{M}_i} = \begin{pmatrix} x_i^1 & \cdots & x_i^{k_i} \\ P(\mathcal{M}_i = x_i^1) & \cdots & P(\mathcal{M}_i = x_i^{k_i}) \end{pmatrix}, \quad (2)$$

with all the possible values x_i^j, and $\sum_{j=1}^{k_i} \mathbb{P}(\mathcal{M}_i = x_i^j) = 1$ by definition.

The cumulative distribution function (cdf) $\mathsf{cdf}_{\mathcal{M}_i}(x) \stackrel{def}{=} P(\mathcal{M}_i \leq x)$ and the inverse cumulative distribution function (1-cdf) $\mathsf{1\text{-}cdf}_{\mathcal{M}_i}(x) \stackrel{def}{=} P(\mathcal{M}_i > x)$ are alternative representations to the pdf. In particular, the 1-cdf outlines the exceedence thresholds as $P(\mathcal{M}_i > x)$.

With $\mathcal{M}_i$ a continue distribution, it is $P(a \leq \mathcal{M}_i \leq b) = \int_a^b \mathsf{pdf}_{\mathcal{M}_i}(y)dy$, $\mathsf{cdf}_{\mathcal{M}_i}(x) = P(\mathcal{M}_i \leq x) = \int_{-\infty}^{x} \mathsf{pdf}_{\mathcal{M}_i}(y)dy$, and $\mathsf{1\text{-}cdf}_{\mathcal{M}_i}(x) = P(\mathcal{M}_i > x) = \int_x^{+\infty} \mathsf{pdf}_{\mathcal{M}_i}(y)dy$.

Worst-case distributions.

In this paper we assume to have worst-case distributions representing message or network elements parameters. By that we mean considering distributions which are greater than or equal to any possible parameter observed [emphirical] distribution in any possible execution condition. See [14] for an insight on the definition of worst-case distributions for the task execution times and the difference between the observed distributions, the worst-case distributions, and approaches for safe worst-case estimates.

The worst-case distributions already accounted for the effect of other system components concurrently executing, e.g. multiple messages accessing the resource to transmit, different queues/switches with variable inputs, etc.. With the worst-case distributions it is possible to assume the statistical independence between system component parameters, thus the independence between components. In the rest of the paper we assume the input $\mathcal{M}$ as discrete worst-case distributions, even if our analysis framework is generic enough to approach continuous inputs. Furthermore, the pC could face the different probabilistic parameter modeling approaches developed so far, [15, 16, 17, 18].

2.1 Probabilistic Flows

A possible representation for a real message flow is throughout a cumulative stochastic process $\mathcal{X}_i(t)$ which counts the amount of resource (bandwidth) requested in a time interval $[0, t)$. Considering the function $R_i(t)$ as the cumulative amount of actual bandwidth requested by m_i up to time t, it is possible to define a probabilistic bound R_i^+ to m_i upper constraining $\mathcal{X}_i$, up to any given probability p_i : $R_i^+(t) \stackrel{def}{=} \sup\{R_i(t) \mid P(\mathcal{X}_i(t) \leq R_i(t)) \leq p_i\}$.

The tuple $(R_i^+(t), p_i)$ represents a probabilistic bound to the m_i trace in the time domain. p_i is a measure of the accuracy/confidence of such bound, since it is the probability

[1]A random variable is a variable whose value is subject to variations due to chance; it can take on a set of possible different values, each with an associated probability.

[2]By calligraphic letters we intend probability distributions, while non-calligraphic letters are for alternatively probabilistic events or deterministic values, with no ambiguity.

that $\mathcal{X}_i(t)$ is upper bounded by the superior $R_i^+(t)$ of the R_is. The case $p_i = 1$ is the deterministic one where $R_i^+(t)$ bounds $\mathcal{X}_i$ without fail.

The arrival curve $\alpha_i(\Delta, \cdot)$ is an abstraction to message flow in the interval domain as an upper bound to all the admissible traces R_i of such stream; $\alpha_i(t-s, \cdot) \;:\; R_i(t) - R_i(s) \leq \alpha_i(t-s, \cdot)$, $\forall s < t$ and $\Delta = t - s$, as stated by the NC [1]. Parameterizing the arrival curve with a variable x, the upper bounding request curve $\alpha_i(\Delta, x) \stackrel{\text{def}}{=} \alpha_i(\Delta, \cdot) + x$ is such that $\alpha_i(t-s, x) - [R_i(t) - R_i(s)] \geq 0$ for all $0 \leq s \leq t$ and $\Delta = t - s$.

It is possible to define the *probabilistic request curve*, inspired by [7, 8, 9], as follow.

Definition 2.1 (Probabilistic request curve). *In a probabilistic framework, the request curve $\alpha_i(\Delta, x)$ of a resource request R_i from message flow m_i is a non-decreasing non-negative function which satisfies*

$$P\left(\alpha_i(t-s, x) - [R_i(t) - R_i(s)] \geq 0\right) \leq p(x) \tag{3}$$

for all $0 \leq s \leq t$, $x \geq 0$ and $\Delta = t - s$. $p(x)$, $p(x) \in [0,1]$ and $p(0) = 0$, specifies the confidence on $\alpha_i(\Delta, x)$ being an upper bound to R_i. The probabilistic request curve of m_i is the couple $\langle \alpha(\Delta, x), p(x) \rangle$.

Definition 2.1 leads to a set of probabilistic request curves $\alpha(\Delta, x)$ from the varying x. Each curve $\alpha(\Delta, x)$ has a confidence $p(x)$ associated of being an upper bound to the message behavior, i.e. the resource demand. The function $p(x) \;:\; \mathbb{N}+ \rightarrow [0,1]$ behaves like a cumulative distribution function: $p(0) = 0$ for the lower bound $\alpha_i(\Delta, 0) = \alpha_i^l$, $p(x^{max}) = 1$ for the deterministic upper bound $\alpha_i(\Delta, 0) = \alpha_i^u$ valid 100% of the time, and the values in between for probabilistic upper bounds. $p(x)$ is a non-decreasing non-negative function.

It is possible to build a distribution $\mathcal{A}$ out of the set of probabilistic request curves obtained varying x. The values of such distribution are the curves $\alpha(\Delta, x)$, while the probabilities associated comes from the $p(x)$s. In particular, it is $\mathsf{cdf}_{\mathcal{A}}(x) = p(x)$. $\mathcal{A}$ is such that:

$$\mathcal{A} = \left(\begin{array}{c} \alpha(\Delta, x) \\ P(\alpha(\cdot, x)) = \mathsf{pdf}_{\mathcal{A}}(x), \end{array} \right), \tag{4}$$

where $\alpha(\Delta, x)$ are the distribution values or we can think about the values of $\mathcal{A}$ as the indexes x as there is the unique association $x \rightarrow \alpha(\Delta, x)$. The probability threshold $\mathsf{cdf}_{\mathcal{A}}(x)$ represents the accuracy/confidence of the $\alpha(\Delta, x)$ bound. Equivalent representation to $\mathcal{A}$ is the couple $(\alpha(\Delta, x), \mathsf{cdf}_{\mathcal{A}}(x))$.

As a reminder, the probabilistic approach we are developing keeps the worst-case deterministic condition $\alpha(\Delta, x^*)$ such that $\mathsf{cdf}_{\mathcal{A}}(x^*) = 1$. In case of distributions $\mathcal{A}$ with finite support, the worst-case would be $\alpha(\Delta, x^{max})$; in case of not finite support distributions the worst-case is definable at the limit, or it could be defined the worst-cases at confidences very close to 1, e.g. $\langle \alpha(\Delta, x^*), \mathsf{cdf}_{\mathcal{A}}(x^*) = 1 - 10^{-30} \rangle$ or $\langle \alpha(\Delta, x^*), \mathsf{cdf}_{\mathcal{A}}(x^*) = 1 - 10^{-50} \rangle$. In this paper we assume to have discrete and finite support input distributions $\mathcal{M}$s, hence discrete and finite support $\mathcal{A}$s.

2.1.1 Probabilistic Bounding Correspondence

It is possible to compute the probabilistic request curves from probabilistic representations like Equation (2) for the message i) inter-arrival times (the minimum inter-arrival time T) with $\mathcal{T}_i$ the distribution representing inter-arrivals variability for m_i, and ii) the message payload C, with $\mathcal{C}_i$ the distribution representing payload variability that m_i could have.

Probabilistic Inter-Arrivals.

With $\mathcal{T}_i$, the set of probabilistic request curve of m_i comes from Inequality (3) and Condition (4).

$$\alpha_i^T(\Delta, x) \stackrel{\text{def}}{=} \begin{cases} \alpha_i^{l,T}(\Delta) & \text{if } x = 0, T = T^{max} \\ \lceil \frac{\Delta}{T_i^x} \rceil \cdot C_i & \text{if } x \in [1, l_i) \\ \alpha_i^{u,T}(\Delta) & \text{if } x \geq \ell_i, T = T^{min}, \end{cases} \tag{5}$$

where x is the index through which differentiate the bounding curves and the probability thresholds associated. For any interval of size Δ, each curve in this set upper bounds a certain percentage of message m_i activations depending on the value assigned to parameter x.

The associated distribution $\mathcal{A}_i^T$ is defined such that:

$$\mathsf{cdf}_{\mathcal{A}_i^T}(x) \stackrel{\text{def}}{=} \begin{cases} 0 & \text{if } x = 0 \\ \sum_{j=0}^{x} \mathsf{pdf}_{\mathcal{T}_i}(T_i^j) & \text{if } x \in [1, l_i) \\ 1 & \text{if } x \geq l_i \end{cases} \tag{6}$$

The $\mathsf{pdf}_{\mathcal{T}_i}$ is also the pdf of the probabilistic curve distribution, thus $\mathsf{pdf}_{\mathcal{A}_i} = \mathsf{pdf}_{\mathcal{T}_i}$. $(\alpha_i^T, \mathsf{cdf}_{\mathcal{A}_i^T})$ includes also the lower bounding curve $\alpha_i^{l,T}(\Delta)$ for consistency reasons: $\alpha^{l,T}$ it the only one with a probability of upper bounding the message behavior equal to 0. $\alpha_i^{u,T}(\Delta)$ is the deterministic upper bound 100% true.

Lemma 2.2 (Bounds from Inter-arrivals, [10]). *Let $\langle \alpha_i^T(\Delta, x), \mathsf{cdf}_{\mathcal{A}_i^T}(x) \rangle$ be defined as in Expressions (5) and (6), then $\alpha_i^T(\Delta) = \lceil \frac{\Delta}{T_i^x} \rceil \cdot C_i$ upper bounds all the request curves of τ_i, in any interval of length Δ such that $T_i \geq T_i^x$ with a probability $\mathsf{cdf}_{\mathcal{A}_i^T}(x) = \mathsf{cdf}_{\mathcal{T}_i}(x)$.*

Probabilistic Payloads/Lengths.

With the pdf $\mathcal{C}_i$ describing the message payload, it is possible to define the following set of request curves relative to the variable payload m_i, with x the index of the probabilistic bounding curves.

$$\alpha_i^C(\Delta, x) \stackrel{\text{def}}{=} \begin{cases} \alpha_i^{l,C}(\Delta) & \text{if } x = 0, C = C^{min} \\ \lceil \frac{\Delta}{T_i} \rceil \cdot C_i^x & \text{if } x \in [1, k_i) \\ \alpha_i^{u,C}(\Delta) & \text{if } x \geq k_i, C = C^{max} \end{cases} \tag{7}$$

For any interval of size Δ, each curve in this set upper bounds a certain percentage of m_i with the associated cdf defined as follows.

$$\mathsf{cdf}_{\mathcal{A}_i^C}(x) \stackrel{\text{def}}{=} \begin{cases} 0 & \text{if } x = 0 \\ \sum_{j=0}^{x} \mathsf{pdf}_{\mathcal{C}_i}(C_i^j) & \text{if } x \in [1, k_i) \\ 1 & \text{if } x \geq k_i \end{cases} \tag{8}$$

The set of probabilistic arrival curves $\left(\alpha_i^C(\Delta, x), \mathsf{cdf}_{\mathcal{A}_i^C}\right)$ defined with Equation (7) and Equation (8) represents the different bounding curves obtained varying the message payload. Within $(\alpha_i^C, \mathsf{cdf}_{\mathcal{A}_i^C})$ there is the lower bounding curve $\alpha_i^{l,C}(\Delta, \cdot)$ for consistency reasons being the only certain not to upper bound any message behavior equal to 0. $\alpha_i^{u,C}(\Delta)$ is the deterministic upper bound 100% true.

LEMMA 2.3 (BOUNDS FROM PAYLOADS, [10]). *Let $\langle \alpha_i^C(\Delta, x), cdf_{\mathcal{A}_i^C} \rangle$ be defined as in Expressions (7) and (8), then $\alpha_i^C(\Delta) = \lceil \frac{\Delta}{T_i} \rceil \cdot C_i^x$ upper bounds all the request curves of τ_i such that $C_i \leq C_i^x$ in any interval of length Δ. The probability threshold is $cdf_{\mathcal{A}_i^C}(x) = cdf_{\mathcal{C}_i}(x)$.*

Figure 3(a) shows an example of probabilistic request curve obtained varying the index parameter x.

EXAMPLE 2.4. *Let $m_i = (\mathcal{C}_i, \mathcal{T}_i)$ be a periodic probabilistic message where $\mathcal{C}_i = \begin{pmatrix} 2 & 3 & 4 \\ 0.5 & 0.4 & 0.1 \end{pmatrix}$ is the probabilistic payload which could vary from one instance to another, and $\mathcal{T}_i = \begin{pmatrix} 15 & 12 & 10 \\ 0.1 & 0.2 & 0.7 \end{pmatrix}$ is the probabilistic inter-arrival time of the message instances. The probabilistic resource request $\mathcal{A}_i$ is represented by Figures 1 and 2 describing the upper bounding curves of m_i and the distribution, respectively.*

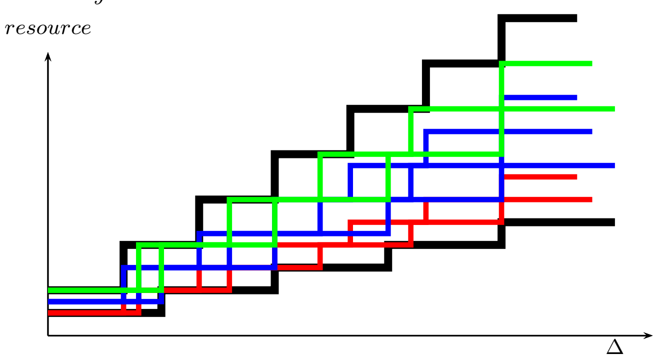


Figure 1: Probabilistic curve: ordered multiple curves $\alpha_i(\Delta, x)$.

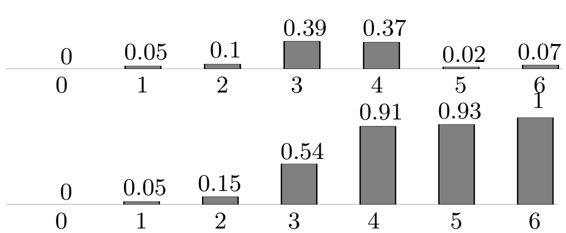


Figure 2: Distribution function $\mathbf{pdf}_{\mathcal{A}_i}(x)$ and $\mathbf{cdf}_{\mathcal{A}_i}(x)$ indexed by x.

It is worthy to note that the probabilistic modeling of message flows (Equation (4), (5),and/or (7)) is straightforward applicable to the characterization of virtual links with AFDX networks.

2.2 Probabilistic Resource Abstraction

For generalization purposes we can follow the same reasoning for arrival curves, i.e. the resource curves, with the service curves, i.e. the request curves. This way, we are capable of probabilistically modeling resource provisioning within networks.

Naming $S(t)$ the total amount of resource provided at time t, the resource provisioning is characterized by considering lower bounding service curve β in the interval domain, namely resource curves. $\beta(\Delta, \cdot) \equiv \beta^u(\Delta, \cdot)$ abstracts the resource provisioning; with $\Delta = t - s$ it is $S(t) - S(s) \geq \beta(t - s, \cdot) \forall s < t$. The resource provided can be modeled according to some cumulative stochastic process $\mathcal{Y}(t)$, and the probabilistic resource bounds is $S(t)^- = \inf\{S(t) \mid \mathbb{P}(\mathcal{Y}(t) \geq S(t)) \leq p\}$.

Then parameterizing the resource provisioning function with a variable y, it follows that for all $0 \leq s \leq t$ and $\Delta = t - s$, the lower bounding service curve $\beta(t - s, y) \stackrel{\text{def}}{=} \beta(\Delta, \cdot) - y$ is such that $[S(t) - S(s)] - \beta(t - s, \cdot) \geq y$. Thus we can define the *probabilistic resource curve* by the definition of bounding probability, [7, 8, 9].

DEFINITION 2.5 (PROBABILISTIC RESOURCE CURVE). *The resource curve $\beta(\Delta, y)$ of a resource provisioning S is a non-decreasing non-negative function which satisfies*

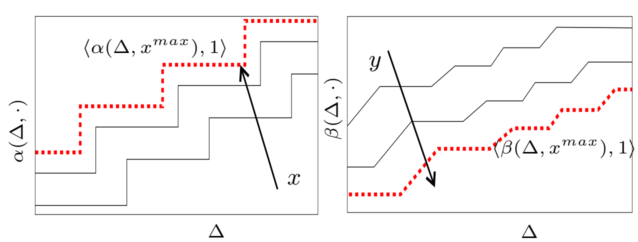
$$P\left([S(t) - S(s)] - \beta(t - s, \cdot) + y \geq 0\right) \leq p(y) \tag{9}$$

for all $0 \leq s \leq t$, $y \geq 0$ and $\Delta = t - s$. $p(y)$ is the probability associated to $\beta(\Delta, y)$. The probabilistic resource curve is representable as $\langle \beta^l(\Delta, y), p(y) \rangle$.

With the notion of probabilistic curve for resource provisioning, $\langle \beta(\Delta, y), \mathsf{cdf}_{\mathcal{B}}(y) \rangle$ we have a set of bounding curves $\beta(\Delta, y)$, each with the changing y and a probability associated (the confidence of being a lower bound). $\mathsf{cdf}_{\mathcal{B}}(y)$ such that $\mathsf{cdf}_{\mathcal{B}}(y) = \mathbb{P}(\mathcal{B} \leq y)$; see Figure 3(b) there is as an example of probabilistic resource provisioning curve. The probability distribution related to $(\beta(\Delta, y), \mathsf{cdf}_{\mathcal{B}}(y))$ is:

$$\mathcal{B} = \begin{pmatrix} \beta(\Delta, y) \\ P(\beta(\cdot, y)) = \mathsf{pdf}_{\mathcal{B}}(y), \end{pmatrix}, \tag{10}$$

with $\beta(\Delta, y)$ the values of $\mathcal{B}$ and $\mathsf{pdf}_{\mathcal{B}}(y)$ is the confidence on $\beta(\Delta, y)$ being a lower bound. Equivalent representation to $\mathcal{B}$ is the couple $(\beta(\Delta, y), \mathsf{cdf}_{\mathcal{B}}(y))$.



(a) Probabilistic request curve: the probability bounding flow resource requests decreases as the quality of the upper bound decreases, $\forall\ x_i \leq x_j$, $\alpha(\Delta, x_i) \leq \alpha(\Delta, x_j)$, $\mathsf{cdf}_{\mathcal{A}}(x_i) \leq \mathsf{cdf}_{\mathcal{A}}(x_j)$.

(b) Probabilistic resource curve: the probability of bounding the resource provisioning decreases as the quality of the lower bound decreases, $\forall\ y_i \leq y_j$, $\beta(\Delta, y_i) \geq \beta(\Delta, y_j)$ and $\mathsf{cdf}_{\mathcal{B}}(y_i) \leq \mathsf{cdf}_{\mathcal{B}}(y_j)$.

Figure 3: Probabilistic request and resource curves.

In Figure 3(b) an example of the sets of probabilistic resource curves where each curve has a probability/confidence associated. The probability threshold $\mathsf{cdf}_{\mathcal{B}}(y)$ describes the accuracy of the β bound. $\beta(\Delta, x^{max})$ with $\mathsf{cdf}(x^{max}) = 1$ is the guaranteed upper bound (100% sure) to the resource provisioning,[4, 10, 1]. Please note that for the service we need to consider lower bounding curves; this guarantees the safety of the analysis approach whenever comparing requests and resources, like for [4, 10, 1].

3. PROBABILITY COMPOSITION

Whenever using probabilistic models for message flows, the relationship between messages becomes statistical, in particular it becomes *statistical dependence*. The joint probability, which expresses the composition between random variables, is affected by the degree of dependence between random variables. The input worst-case distributions assumption guarantees independence between the parameters as well as independence between the probabilistic bounding curves. Thus, for a couple of message flow distributions $\mathcal{A}_i$ and $\mathcal{A}_j$, the joint probability is $\mathsf{pdf}_{\mathcal{A}_i,\mathcal{A}_j} = \mathsf{pdf}_{\mathcal{A}_i} \otimes \mathsf{pdf}_{\mathcal{A}_j}$, as the conditional probability $\mathsf{pdf}_{\mathcal{A}_i|\mathcal{A}_j} = \mathsf{pdf}_{\mathcal{A}_i}$. The $\otimes$ operator is the convolution between distributions. In case of no statistical independence it would be $\mathsf{pdf}_{\mathcal{A}_i,\mathcal{A}_j} = \mathsf{pdf}_{\mathcal{A}_i|\mathcal{A}_j} \otimes \mathsf{pdf}_{\mathcal{A}_j} = \mathsf{pdf}_{\mathcal{A}_j|\mathcal{A}_i} \otimes \mathsf{pdf}_{\mathcal{A}_i}$ which is manageable knowing the conditional relationship between $\mathcal{A}_i$ and $\mathcal{A}_j$, for example with the copula convolution [19].

3.1 Probabilistic Network Components

In both NC and RTC there exist output curves to a system component. In particular, with networks we could name outputs as the executed/processed flow $\alpha^{'}$, and the unused resource $\beta^{'}$ for processing such flows. The probabilistic output curves can be inferred by applying the relationship between the arrival and service inputs, [6, 7, 8, 9].

The probabilistic output request curve (output arrival curve) is defined as the curve $(\alpha^{'}(\Delta, x), \mathsf{cdf}_{\mathcal{A}'}(x))$ with $\alpha^{'}(\Delta,\cdot) \stackrel{\text{def}}{=} \alpha\overline{\oslash}\beta(\Delta,\cdot)$[3] and the threshold probability $\mathsf{pdf}_{\mathcal{A}'}(\cdot) \stackrel{\text{def}}{=} \mathsf{pdf}_{\mathcal{A}} \otimes \mathsf{pdf}_{\mathcal{B}}(\cdot)$ [4]:

$$\mathcal{A}_i' \stackrel{\text{def}}{=} (\alpha_i' \stackrel{\text{def}}{=} \alpha_i\overline{\oslash}\beta(\Delta,\cdot), \mathsf{cdf}_{\mathcal{A}_i'}(x) \stackrel{\text{def}}{=} \mathsf{cdf}_{\mathcal{A}_i} \cdot \mathsf{cdf}_{\mathcal{B}}(x)). \quad (11)$$

The probabilistic curve bounds the cumulative amount of resource processed up to time t, $R'(t)$, as follows:

$$P\left(\alpha_i^{'}(\Delta, x) - [R_i'(t) - R_i'(s)] \geq 0\right) \leq \mathsf{cdf}_{\mathcal{A}'}(x). \quad (12)$$

The unused resource is passed to other parts of the system according to a specific strategy. The probabilistic version of the residual curve $\langle\beta^{'}(\Delta, y), \mathsf{cdf}_{\mathcal{B}'}(y)\rangle$ is such that $\beta^{'}(\Delta,\cdot) \stackrel{def}{=} \beta \oslash \alpha(\Delta,\cdot)$[5], and $\mathsf{pdf}_{\mathcal{B}'}(\cdot) \stackrel{def}{=} \mathsf{pdf}_{\mathcal{A}} \otimes \mathsf{pdf}_{\mathcal{B}}(\cdot)$:

$$\mathcal{B}_i' \stackrel{\text{def}}{=} (\beta' \stackrel{\text{def}}{=} \beta \oslash \alpha(\Delta,\cdot), \mathsf{cdf}_{\mathcal{B}'}(x) \stackrel{\text{def}}{=} \mathsf{cdf}_{\mathcal{A}_i} \cdot \mathsf{cdf}_{\mathcal{B}}(x)), \quad (13)$$

with the output bounds such that:

$$P\left([S'(t) - S'(s)] - \beta'(\Delta, y) \geq 0\right) \leq \mathsf{cdf}_{\mathcal{B}'}(y). \quad (14)$$

Let us add few remarks on Equation (12), Equation (14), Equation (11), and Equation (13). The probabilistic outputs $(\alpha', \mathsf{cdf}_{\mathcal{A}'})$ and $(\beta', \mathsf{cdf}_{\mathcal{B}'})$ are obtained assuming independence between inputs $(\alpha, \mathsf{cdf}_{\mathcal{A}})$ and $(\beta, \mathsf{cdf}_{\mathcal{B}})$. This is reasonable since there should not be dependence between message flows α and resource provisioning β. Due to independence, the joint probability, as the probability of having both inputs present, comes from the convolution between the input distributions. In case of dependence between flows and resource provisioning, the output probabilities are the joint probabilities in the general form, $\mathsf{pdf}_{\mathcal{A}'} \equiv \mathsf{pdf}_{\mathcal{B}'} = \mathsf{pdf}_{\mathcal{A}_i,\mathcal{B}} = \mathsf{pdf}_{\mathcal{A}_i|\mathcal{B}} \otimes \mathsf{pdf}_{\mathcal{B}}$, and computable once characterized the conditional effect $\mathsf{pdf}_{\mathcal{A}_i|\mathcal{B}}$ and $\mathsf{pdf}_{\mathcal{B}|\mathcal{A}_i}$ of one to the other.

Within a probabilistic framework there exist probabilistic components for networks, where probabilistic interfaces are described with the tuple $((\alpha, \mathsf{cdf}_{\mathcal{A}}), (\beta, \mathsf{cdf}_{\mathcal{B}}), (\alpha', \mathsf{cdf}_{\mathcal{A}'}), (\beta', \mathsf{cdf}_{\mathcal{B}'}))$. The generic network component and the input/output bounds forming the probabilistic interface to the component are illustrated in Figure 4.

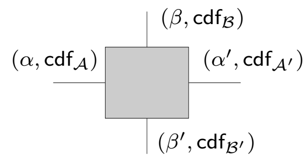


Figure 4: Interface of a probabilistic network component.

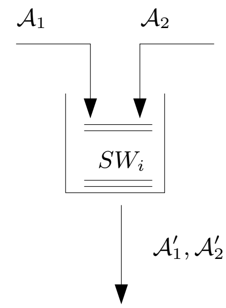


Figure 5: FIFO network component (e.g. AFDX switch) with two input flows.

3.1.1 FIFO

Equation (12) can be applied for modeling FIFO queue components like the AFDX switches SW, Figure 5 for an example. In there, the input interface has only message flows, since the queue applies the whole bandwidth resource assigned, $\beta(\Delta,\cdot) = \Delta$. In order to compute resource output, the resource provided to each flow have to be known. Considering the case of two input probabilistic flows $\mathcal{A}_1$ and $\mathcal{A}_2$, the resource provided to $\mathcal{A}_1$ is $\mathcal{B}_1$, and is computed as the residual unused by $\mathcal{A}_2$, such that:

$$\begin{aligned}\beta_1(\Delta, k) &= \beta(\Delta, y) - \alpha_2(\Delta, x)\\ \mathsf{cdf}_{\mathcal{B}_1}(k) &= \mathsf{cdf}_{\mathcal{A}_2}(x) \cdot \mathsf{cdf}_{\mathcal{B}}(y).\end{aligned} \quad (15)$$

Generalizing to more than two input flows, it is $\mathcal{B}_j$ such that:

$$\begin{aligned}\beta_j(\Delta, k) &= \beta(\Delta, y) - \alpha_{\overline{j}}(\Delta, x)\\ \mathsf{cdf}_{\mathcal{B}_j}(k) &= \mathsf{cdf}_{\mathcal{A}_{\overline{j}}}(x) \cdot \mathsf{cdf}_{\mathcal{B}}(y).\end{aligned} \quad (16)$$

The probabilistic curves can be cumulated as:

$$\mathcal{A}_{\overline{j}}(x) = \otimes_{k\neq j}\mathcal{A}_k(x),$$

such that $\alpha_{\overline{j}}(\Delta, x) = \sum_{k\neq j}\alpha_k(\Delta, x)$ and $\mathsf{pdf}_{\mathcal{A}_{\overline{j}}}(x) = \otimes_{k\neq j}\mathsf{pdf}_{\mathcal{A}_k}(x)$, to have the total resource request from a number of flow messages, thus compute the residual services.

From Equation (16) the probabilistic interface to FIFO switch is $((\alpha_1, \mathsf{cdf}_{\mathcal{A}_1}), \ldots (\alpha_n, \mathsf{cdf}_{\mathcal{A}_n}), (\alpha_1', \mathsf{cdf}_{\mathcal{A}_1'}), \ldots (\alpha_n', \mathsf{cdf}_{\mathcal{A}_n'}))$.

3.2 Probabilistic Performances

With the [deterministic] NC it exists the notion of backlog BL which comes from the combination of the arrival and service curves:

$$B \leq v(\alpha, \beta) = sup_{\Delta\geq 0}\{\alpha(\Delta) - \beta(\Delta + \tau)\}\}. \quad (17)$$

[3]As defined in RTC [4] and by the notion of max-plus deconvolution $\overline{\oslash}$, [1].

[4]$\otimes$ is the convolution between functions in the classical algebra; $\otimes$ is the min-plus convolution for RTC curves, and $\overline{\otimes}$ is the max-plus convolution.

[5]As defined in RTC [4] and by the notion of min-plus deconvolution $\oslash$, [1].

BL is the vertical distance between the resource demand and the resource provisioning curves. The delay DEL in NC is defined as:

$$D \leq h(\alpha, \beta) = sup_{\Delta \geq 0}\{inf\{\tau \geq 0 \ / \ \alpha(\Delta) \leq \beta(\Delta + \tau)\}\}, \quad (18)$$

being the horizontal distance between α and β. Both backlog and delay defines the performance of a network component.

With a probabilistic framework, due to the probabilistic nature of α and β, it is possible to define performance metrics like delay and backlog as random variables. The probabilistic backlog is the couple $(BL(k), p(k))$ such that:

$$BL(k) = sup_{\Delta \geq 0}\{\alpha(\Delta, x) - \beta(\Delta + \tau, y)\}\}$$
$$p(k) = \mathsf{cdf}_{\mathcal{A}}(x) \cdot \mathsf{cdf}_{\mathcal{B}}(y),$$

with $k = (x, y)$. The probabilistic backlog is then a distribution $\mathcal{BL} \equiv (BL(k), p(k))$ for which:

$$\mathsf{pdf}_{\mathcal{BL}} = \begin{pmatrix} BL^0 & \cdots & BL^k \\ P(\mathcal{BL} = BL^0) & \cdots & P(\mathcal{BL} = BL^k) \end{pmatrix}, \quad (19)$$

where the BL^k are the $BL(x, y)$ values from $\alpha(\Delta, x)$ and $\beta(\Delta, y)$.

The probabilistic delay $\mathcal{DEL} = \equiv (DEL(k), p(k))$ is:

$$DEL(k) = sup_{\Delta \geq 0}\{inf\{\tau \geq 0 \mid \alpha(\Delta) \leq \beta(\Delta + \tau)\}\}$$
$$p(k) = \mathsf{cdf}_{\mathcal{A}}(x) \cdot \mathsf{cdf}_{\mathcal{B}}(y),$$

with the delay distribution $\mathcal{DEL}$:

$$\mathsf{pdf}_{\mathcal{DEL}} = \begin{pmatrix} DEL^0 & \cdots & DEL^k \\ P(\mathcal{DEL} = DEL^0) & \cdots & P(\mathcal{DEL} = Del^k) \end{pmatrix}, \quad (20)$$

where the value $DEL^k = DEL(x, y)$ comes from $\alpha(\Delta, x)$ and $\beta(\Delta, y)$.

3.2.1 Performance Composition

The end-to-end probabilistic delay of a message flow is the the joint distribution among the delay distributions of the components crossed by the message flow. The performance of a component-based network can be composed for probabilistic end-to-end delays. In case of independence between delays, the joint probability can be computed as the convolution of the delays:

$$\mathcal{DEL} = \otimes_{SW_i} \mathcal{DEL}_i \quad (21)$$

The independence between components, thus between delays, is the consequence of the flow independence assumption and the use of worst-case distributions. The same could happen to the backlog.

4. SPREADING PROBABILITIES: CASE STUDY

In this section we show how the probabilities combine through network components and the network topology, thus the way the flow relationship propagates within the network. To investigate probabilistic networks we make use of a probabilistic component-based simulator implemented in R[6]. The simulator 1) computes probabilistic curves from input parameter distributions, it 2) computes also the output probabilistic curves for the network components already modeled, and it 3) computes the network component performance combining the probabilistic input curves. The network topology is built by hand by connecting the probabilistic components.

[6]https://www.r-project.org/

For this case study we consider an AFDX network example with 4 flows (α_1, α_2, α_3, and α_3) and three queues (the switches SW_1, SW_2, and SW_3), as in Figure 6. We assume α_1 and α_2 accessing SW_1, the α_1 output of SW_1 $\alpha_1^{(1)}$ and α_3 accessing SW_2, and the α_1 output of SW_2 $\alpha_1^{(2)}$ and α_4 access SW_3. The probabilistic inputs associated are $(\alpha_1(\Delta, x), \mathsf{cdf}_{\mathcal{A}_1}(x))$, $(\alpha_2(\Delta, x), \mathsf{cdf}_{\mathcal{A}_2}(x))$, $(\alpha_3(\Delta, x), \mathsf{cdf}_{\mathcal{A}_3}(x))$, and $(\alpha_4(\Delta, x), \mathsf{cdf}_{\mathcal{A}_4}(x))$. The distributions $\mathcal{A}$s follows the Poisson ($dist^1$) and the Weibull ($dist^2$) discrete and with finite supports laws. The Poisson and the Weibull distributions are applied to represent worst-case distributions for two different behaviors: with small tails (the Poisson) and heavy tails (the Weibull). The distributions are for the input flows and could result from a probabilistic representation of the flow parameters such as inter-arrivals and/or payloads. The relationships between $\mathcal{M}_i$s and $\mathcal{A}_i$s comes from Equation (5), (6), (7), and (8).

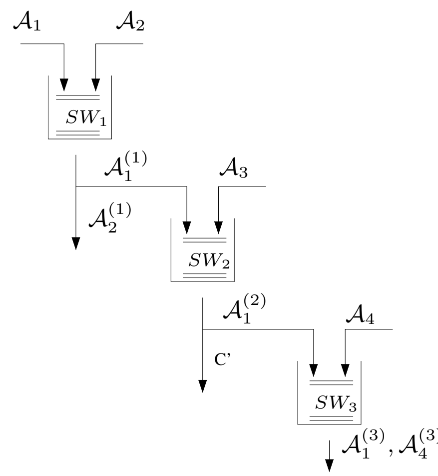


Figure 6: Case study with 3 switches and 4 independent input flows.

Considering the independence between flows, $\mathcal{A}_1 \overline{\triangleright} \mathcal{A}_2$, at the output of SW_1 it is :

$$\mathcal{A}_1^{(1)} = (\alpha_1^{(1)}(\Delta, x), \mathsf{cdf}_{\mathcal{A}_1^{(1)}}(x))$$
$$\mathcal{A}_2^{(1)} = (\alpha_2^{(1)}(\Delta, x), \mathsf{cdf}_{\mathcal{A}_2^{(1)}}(x)),$$

with $\mathsf{pdf}_{\mathcal{A}_1^{(1)}}(x) = \mathsf{pdf}_{\mathcal{A}_2^{(1)}}(x) = \mathsf{pdf}_{\mathcal{A}_1} \otimes \mathsf{pdf}_{\mathcal{A}_2}$, Equation (11).
At the output of SW_2, due to $\mathcal{A}_1 \overline{\triangleright} \mathcal{A}_3$, it is :

$$\mathcal{A}_1^{(2)} = (\alpha_1^{(2)}(\Delta, x), \mathsf{cdf}_{\mathcal{A}_1^{(2)}}(x))$$
$$\mathcal{A}_3^{(2)} = (\alpha_3^{(2)}(\Delta, x), \mathsf{cdf}_{\mathcal{A}_3^{(2)}}(x)),$$

with $\mathsf{pdf}_{\mathcal{A}_1^{(2)}}(x) = \mathsf{pdf}_{\mathcal{A}_3^{(2)}}(x) = \mathsf{pdf}_{\mathcal{A}_1^{(1)}} \otimes \mathsf{pdf}_{\mathcal{A}_3}$, Equation (11).
At the output of SW_3, due to $\mathcal{A}_1 \overline{\triangleright} \mathcal{A}_4$, it is :

$$\mathcal{A}_1^{(3)} = (\alpha_1^{(3)}(\Delta, x), \mathsf{cdf}_{\mathcal{A}_1^{(3)}}(x))$$
$$\mathcal{A}_4^{(3)} = (\alpha_4^{(3)}(\Delta, x), \mathsf{cdf}_{\mathcal{A}_4^{(3)}}(x)),$$

with $\mathsf{pdf}_{\mathcal{A}_1^{(3)}}(x) = \mathsf{pdf}_{\mathcal{A}_4^{(3)}}(x) = \mathsf{pdf}_{\mathcal{A}_1^{(2)}} \otimes \mathsf{pdf}_{\mathcal{A}_4}$, Equation (11).

Figure 7 shows how $\mathcal{A}_1$ propagates throughout the network in case of input distributions following a Poisson law.

The distributions are represented in terms of the index x (the values) and the upper bounding cumulative probabilities. The probability distributions at the switch outputs are the results of Equation (11). Some of the probabilistic curves and their propagation through the SWs are represented in Figure 8. The output curves are computed with Equation (11).

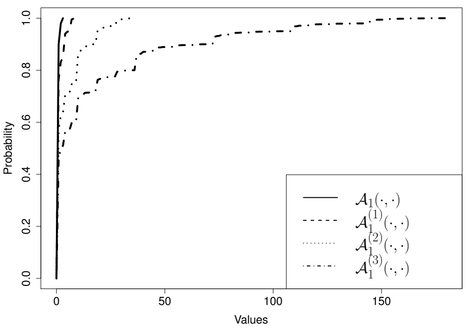
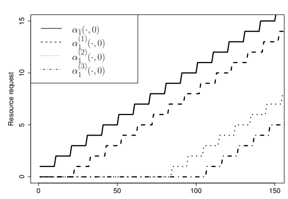
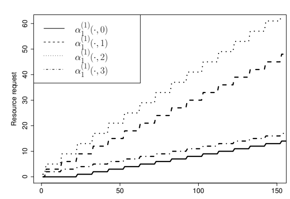


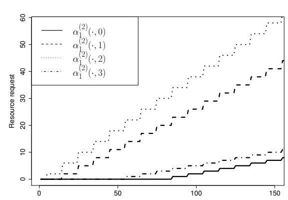
Figure 7: $\mathcal{A}_1$ propagation throughout the network from $dist^1$. cdf representation for the $\mathcal{A}_1$s.



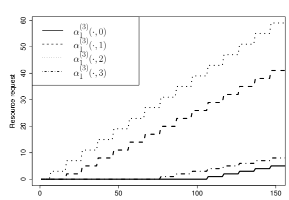
(a) $\alpha_1(\Delta, 0)$ curves throughout all the SWs



(b) $\alpha_1^{(1)}$ curves output of SW_1



(c) $\alpha_1^{(2)}$ curves output of SW_1



(d) $\alpha_1^{(3)}$ curves output of SW_1

Figure 8: Curve propagation with $\mathcal{A}$s following $dist^1$.

Figure 9 is the representation of the $\mathcal{A}_1$ propagation in case of input distributions following the Weibull law.

Figure 10 and Figure 11 depicts respectively the delay and backlog behavior throughout the network. $\mathcal{DEL}_1$ and $\mathcal{BL}_1$ are the delay and backlog distributions of SW_1, $\mathcal{DEL}_2$ and $\mathcal{BL}_2$ are the delay and backlog distributions of SW_2, and $\mathcal{DEL}_3$ and $\mathcal{BL}_3$ are the delay and backlog distributions of SW_3; all of them are from Equation (20) and Equation (19), respectively for the delay and backlogs, in case of input distributions following $dist^1$. $\mathcal{DEL}$ is the convolution of the delays for the end-to-end delay distribution of $\mathcal{A}_1$ traversing the 3 switches, Equation (21).

The delays and backlog counterparts from $dist^2$ are represented in Figure 12 and Figure 13.

5. CONCLUSION

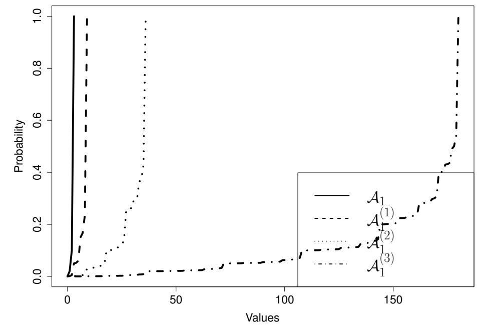


Figure 9: $\mathcal{A}_1$ propagation throughout the network from $dist^2$. cdf representation for the $\mathcal{A}_1$s.

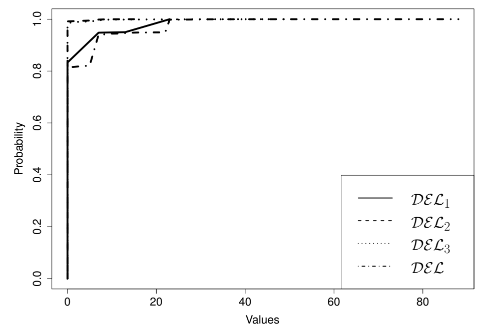


Figure 10: Delays with $dist^1$.cdf representation for the $\mathcal{DEL}$s.

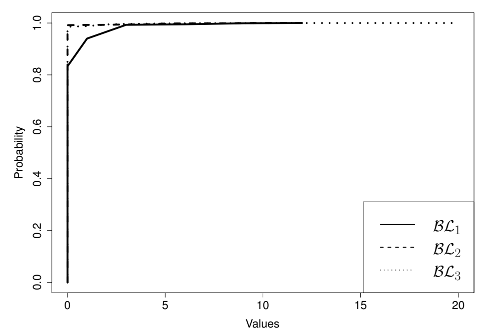


Figure 11: Backlogs with $dist^1$. cdf representation for the $\mathcal{BL}$s.

With this paper we have applied probabilities to component-based modeling and analysis of timing constrained networks. We have formalized and developed the probabilistic calculus for network analysis with the use of probabilities. The probabilistic Calculus framework face probabilistic networks where the network element parameters are described as distributions. Besides, it models and analyses network elements with probabilistic bounding curves to characterize both the network elements and the whole network behavior. Hard and soft timing guarantees to the network performance are given in terms of performance.

For the future, we want to enhance the probabilistic analysis by tackling with the confidence and the guarantees that

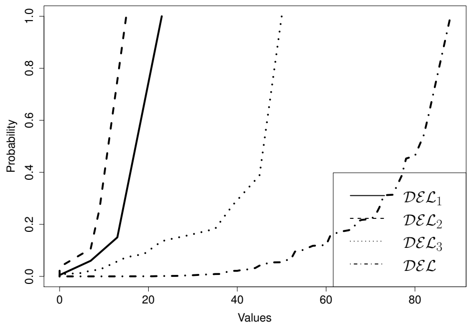


Figure 12: Delays with $dist^2$. cdf representation for the $\mathcal{DEL}$s.

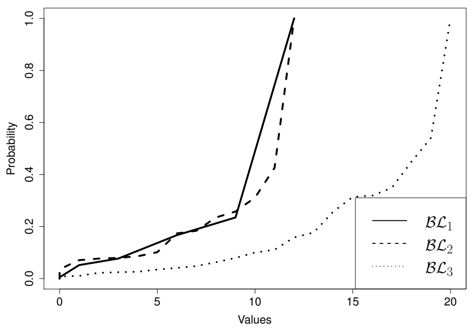


Figure 13: Backlogs with $dist^2$. cdf representation for the $\mathcal{BL}$s.

a probabilistic framework could offer. Furthermore, we want to extend modeling to other network parameters/network elements and complete the probabilistic view to time-constrained networks such as AFDX. Finally, we aim at enhancing the pC for tackling with adaptive network behavior as well as the mixed-critical network behavior.

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